

Subject Code : 35 (NS)

1064953

MATHEMATICS

(Kannada and English Versions)

Time : 3 Hours 15 Minutes]

[Total No. of questions : 50]

[Max. Marks : 100

(English Version)

- Instructions :**
1. The question paper has five Parts A, B, C, D and E. Answer **all** the Parts.
 2. Use the Graph Sheet for the question on Linear Programming in Part-E.

PART - A

Answer all the ten questions :

(10 × 1 = 10)

- 1) Let * be the binary operation on N , given by $a * b = \text{LCM of } a \text{ and } b$. Find $20 * 16$.
- 2) Find the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$.
- 3) Construct a 2×2 matrix, $A = [a_{ij}]$, where elements are given by,

$$a_{ij} = \frac{i}{j}$$
- 4) If A is a square matrix with $|A| = 8$ then find the value of $|AA'|$.
- 5) If $y = \cos \sqrt{x}$, find $\frac{dy}{dx}$.
- 6) Find $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$.
- 7) Define collinear vectors.
- 8) Find the direction cosines of a line which makes equal angles with the positive co-ordinate axes.



9) Define feasible region in a linear programming problem.

10) If A and B are independent events, $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$ then find $P(A \cap B)$.

PART - B

Answer any ten questions :

(10 × 2 = 20)

11) If $f: R \rightarrow R$ defined by $f(x) = 1 + x^2$, then show that f is neither 1-1 nor onto.

12) Show that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x$, $\frac{1}{\sqrt{2}} \leq x \leq 1$.

13) Solve the equation $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$, ($x > 0$).

14) Find the values of k , if area of triangle is 4 sq. units and vertices are $(k, 0)$, $(4, 0)$ and $(0, 2)$ using determinant.

15) If $ax + by^2 = \cos y$, find $\frac{dy}{dx}$.

16) Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8$, $x \in [-4, 2]$.

17) Find the approximate change in the volume of a cube of side x metres caused by increasing the side by 3%.

18) Integrate $\frac{\tan^4 \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}}$ with respect to x .



- 19) Evaluate $\int_0^{2/3} \frac{dx}{4+9x^2}$.
- 20) Find the order and degree of the differential equation $\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} - \sin^2 y = 0$.
- 21) Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively, in the ratio 2 : 1
- Internally.
 - Externally.
- 22) Find the area of the parallelogram whose adjacent sides are determined by the vector $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.
- 23) Find the vector and Cartesian equation of the line that passes through the points $(3, -2, -5)$ and $(3, -2, 6)$.
- 24) Find the probability distribution of number of heads in two tosses of a coin.

PART - C

Answer any ten questions :

(10 × 3 = 30)

- 25) Show that the relation R in \mathbb{R} (set of real numbers) is defined as $R = \{(a, b) : a \leq b\}$ is reflexive and transitive but not symmetric.

- 26) Write $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $x \neq 0$ in the simplest form.



- 27) If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if $AB = BA$.
- 28) Differentiate $(\log x)^{\cos x}$ with respect to x .
- 29) Differentiate $\sin^2 x$ with respect to $e^{\cos x}$.
- 30) Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.
- 31) Evaluate : $\int \frac{2x}{x^2 + 3x + 2} dx$.
- 32) Evaluate : $\int e^x \sin x dx$.
- 33) Find area of the region bounded by the curve $y^2 = 4x$, y -axis and the line $y = 3$.
- 34) Form the differential equation of the family of circles having centre on y -axis and radius 3 units.
- 35) Find x , such that the four points $A(3, 2, 1)$, $B(4, x, 5)$, $C(4, 2, -2)$ and $D(6, 5, -1)$ are coplanar.
- 36) Three vectors \vec{a} , \vec{b} and \vec{c} satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, evaluate $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $|\vec{a}| = 1$, $|\vec{b}| = 4$ and $|\vec{c}| = 2$.

- 37) Find the shortest distance between the lines $\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$
and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$.

- 38) Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

PART - D

Answer any six questions :

(6 × 5 = 30)

- 39) Let $f: N \rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: N \rightarrow S$, where S is the range of f , is invertible. Find the inverse of f .
- 40) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ prove that $A^3 - 6A^2 + 7A + 2I = 0$.
- 41) Solve the following system of linear equations by matrix method.

$$x - y + 2z = 1$$

$$2y - 3z = 1$$
and $3x - 2y + 4z = 2$.
- 42) If $y = (\tan^{-1} x)^2$. Show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$.
- 43) The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When $x = 8$ cm and $y = 6$ cm, find the rate of change of
i) The perimeter and
ii) The area of the rectangle.



- 44) Find the integral of $\sqrt{x^2 - a^2}$ with respect to x and hence evaluate $\int \sqrt{x^2 - 8x + 7} dx$.
- 45) Using integration find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.
- 46) Solve the differential equation $\cos^2 x \frac{dy}{dx} + y = \tan x$ $\left(0 \leq x < \frac{\pi}{2}\right)$.
- 47) Derive the equation of a plane perpendicular to a given vector and passing through a given point both in vector and Cartesian form.
- 48) The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs
- none
 - not more than one
 - more than one
- will fuse after 150 days of use.

PART - E

Answer any one question :

(1 × 10 = 10)

- 49) a) Prove that $\int_0^a f(x) dx = \int_0^a f(a-x) dx$ and hence evaluate
- $$\int_0^{\pi/2} (2 \log \sin x - \log \sin 2x) dx. \quad (6)$$

b) Show that :

$$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy + yz + zx). \quad (4)$$

50) a) Minimize and Maximize $z = 600x + 400y$

Subject to the constraints :

$$x + 2y \leq 12$$

$$2x + y \leq 12$$

$$4x + 5y \geq 20 \text{ and } x \geq 0, y \geq 0 \text{ by graphical method.}$$

(6)

b) Find the value of k , if

$$f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & x \neq \frac{\pi}{2} \\ 3, & x = \frac{\pi}{2} \end{cases}$$

is continuous at $x = \frac{\pi}{2}$.

(4)